Stopping power of hot QCD plasma

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The partonic energy loss has been calculated taking all the $2 \to 2$ processes into account. Relative importance of individual channel is clearly revealed. We also discuss subtleties related to the identical final state partons. The estimated collisional loss is significantly large. In addition, we present closed form formulas for both the collision probabilities and stopping power (dE/dx).

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The partonic energy loss in a QCD plasma has received significant attention in recent years. Experimentally, the partonic energy loss can be probed by measuring the high p_T hadrons emanating from ultra-relativistic heavy ion collisions. This idea was first proposed by Bjorken [1] where 'ionization loss' of the quark and gluons in a QCD plasma was estimated. In fact, the 'stopping power' (dE/dx) of the plasma is proportional to $\sqrt{\epsilon}$, where, ϵ is the energy density of the partonic medium. Therefore, by measuring various high p_T observables one can probe the initial parton density [1].

Hard partons, injected into hot QCD medium, can dissipate energy in two ways, *viz.*, by two body collisions or *via* the bremsstrahlung emission of gluons, commonly referred to as collisional and radiative loss respectively. For electromagnetic processes, it is well known that at large energies, radiation losses are much higher than the collisional loss. At low energy, however, the latter is more dominant. In fact, there is a critical energy E_c, at which, both the processes contribute equally [2–4]. In QCD plasma, to our knowledge, such estimation of E_c is not known yet. Present work is an attempt towards this direction. Obviously this requires complete treatment of both $2 \to 2$ and $2 \to 3$ (or higher order) processes. While significant progress has been made over the past decade to estimate bremsstrahlung induced partonic energy loss [5–13], collisional loss, as we uncover, begs further attention. In the original proposal, Bjorken actually considered only $2 \to 2$ loss [1], however, this and subsequent papers observe a marginal value, $(dE/dx)_{\rm coll} \ll 1$ GeV/fm [1,5,14–16]. This is much less than what we report in this letter. It might be mentioned that calculations reported in [1,14] were restricted only to the t channel processes, thereby, excluding the interference and exchange terms, which contribute significantly. They are particularly important for processes having u channel divergences.

In the present work we take all possible diagrams including elastic and inelastic scatterings. Consequently, estimated $(dE/dx)_{\rm coll}$, is higher than reported before. In fact, the two body compton like scattering, proves to be quite efficient in transferring energy into the plasma. Furthermore, unlike heavy flavor energy loss [16] in a plasma (with light constituents), light quark involves subtleties related to the processes having identical final state species. This has not been considered earlier. In addition, we also evaluate explicitly the collisional loss of gluon energy. For this, $gg \to gg$ and $qg \to qg$ are found to be most important.

To calculate the stopping power of QCD plasma arising out of two body scatterings, we introduce a formalism along the line similar to what is employed to study cosmic ray showers [2,3]. Therefore, we define a differential collision probability, $\Theta(E,E')dE'dx$ which represents the probability of a parton with energy E to transfer an amount of energy between E' and $E'+dE'$ to a plasma constituent in traversing a thickness dx. Energy loss can be obtained by convoluting $\Theta(E,E')dE'$ with the energy transfer (E') for each processes which generically is given by,

$$\frac{dE}{dx} = \int_{E_{\rm min}}^{E_{\rm max}} E'\Theta(E,E')dE'. \qquad (1)$$

In the above equation, $E_{\rm max}$ is the maximum energy transfer, while $E_{\rm min}$ is a cut off used to regulate infrared divergence related to the usual small angle limit. However, in case of energy loss calculation $\int \frac{d\theta}{\theta^3}$ divergence that appears in the cross section becomes softer as schematically $dE/dx \sim n\sigma E'$, where, $E'/E = \frac{1}{2}(1-\cos\theta) \sim \theta^2/4$, tames the divergence. This cut-off procedure can be avoided by incorporating appropriate screening effects [15,16]. A complete treatment of this singularity in the context of heavy quark energy loss both for soft and hard momentum transfer within hard thermal loop resummation scheme has been discussed in [16]. However, as the motivation of the present work is to unravel the relative contribution of each processes, to deal with the small angle divergence, we take the approach of Ref. [1]. It might be mentioned that for coulomb like scattering, $\Theta(E,E') \propto 1/E'^2$ and therefore $dE/dx \propto \int \frac{dE'}{E'}$. This, evidently, is divergent and gives the logarithmic dependence of dE/dx [1]. In the subsequent sections, we present explicit expressions of $\Theta(E,E')dE'$ for various processes. Corresponding QED results are also derived for comparison [2,3]. The quantities E and E' are defined in the laboratory frame. It should be remarked that, for dE/dx, the large angle scatterings are important as it involves large energy transfer [17]. The small angle scatterings, nevertheless, are also important because of their enhanced rates caused by the infrared singularities. Hence we retain contributions from all angles.

Let us first consider propagation of a hard quark through a QCD plasma. The collisions which would contribute to its energy loss are $qq \to qq$, $qq' \to qq'$, $q\bar{q}' \to q\bar{q}'$, $qg \to qg$ and $q\bar{q} \to q\bar{q}, gg, q'\bar{q}'$. In the above processes primes indicate different flavors. It might be mentioned that in a baryon free region, *i.e.*, in absence of a net baryonic chemical potential, quark and anti-quark energy loss will be the same. Therefore, we do not treat them separately. Likewise, gluonic energy loss can also be calculated. This is expected to be much larger than collisional loss of quarks because of larger gg cross section.

The most dominant process for quark energy loss, as mentioned before, is compton like scattering, *i.e.* $qg \to qg$. Beside the t channel, contributions from additional diagrams are found to be non-negligible.

The differential collision probability for the compton like scattering is as follows:

$$\Theta_{qg\to qg}(E,E')dE' = \frac{\pi\alpha_s^2}{2\omega E^2}\rho_g\big[1 - 2\frac{E}{E'} + 2(\frac{E}{E'})^2 + \frac{4}{9}\{1 - \frac{E'}{E} + \frac{E}{E-E'}\}\big]dE' \quad (2)$$

In the last equation and subsequent expressions, $\rho_{q(g)}$ denotes quark (gluon) density which for a equilibrated plasma can be written as : $\rho_{q,g} = \gamma_{q,g}\int \frac{d^3k}{(2\pi)^3} f_{q,g}(k)$. Here, $\gamma_{q,g}$ is the appropriate degeneracy factor, $f(k) = \frac{1}{e^{k/T}\pm 1}$, plus or minus sign would correspond to the quark and gluon state, while, ω denotes parton energy in the static limit which can be evaluated using the techniques of finite temperature quantum fields [18].

In Eq. 2, the first three terms come from the t channel and others originate from the exchange diagram and s channel. Evidently, $E/(E-E')$ gives rise to logarithmic enhancement. Thus overall contribution becomes significant if one retains all the possible diagrams. To get the energy loss, one integrates the differential collision probability weighted with the energy transfer as shown in Eq. 1 to yield :

$$(\frac{dE}{dx})_{qg\to qg} = \frac{\pi\alpha_s^2}{2\omega}\rho_g\big[\frac{22}{9}\left(\ln(\frac{2E}{\omega}) + \frac{\omega}{2E}\right) - \frac{13}{18}(\frac{\omega}{2E})^2 + \frac{4}{27}(\frac{\omega}{2E})^3 - \frac{101}{54}\big] \quad (3)$$

In writing the above equation, we have used $E_{\min} = \omega/2$, and $E_{\max} = E$, the energy of the hard parton. It should be noted that the coefficient of the logarithmic term is different from that one obtains by restricting to the t channel alone.

Next we consider Möller type $qq \to qq$ scattering for which the differential collision probability [17] reads,

$$\Theta_{qq\to qq}(E,E')dE' = \frac{4}{9}\frac{\pi\alpha_s^2}{\omega}\rho_q\big[\frac{E^2}{E'^2(E-E')^2} + \frac{\Delta}{E'(E-E')} + \frac{1}{E^2}\big]dE', \quad (4)$$

where, $\Delta = -10/3$. In the last expression if we replace $\frac{2}{9}\alpha_s^2$ by α_{em}^2 and Δ by -2, the electron energy loss due to Möller scattering ensues [2,3]. This reaction deserves special attention as it involves two identical particles in the final state. Therefore, $\Theta(E,E')dE'$, in this case, should be interpreted as the probability of a collision which leaves one parton in the energy state E' and the other in the energy state $E - E'$. To take into account all the possibilities, E' is varied from $\omega/2$ to $E/2$ [2,3]. Similar subtlety is involved for processes like $gg \to gg$ or $q\bar{q} \to gg$ etc. The final expression for $qq \to qq$ is given by

$$\left(\frac{dE}{dx}\right)_{qq\to qq} = \frac{4}{9}\frac{\pi\alpha_s^2}{\omega}\rho_q\big[\ln(2E/\omega) + \Delta\ln 2 - \frac{2E}{2E-\omega} - \frac{1}{2}(\frac{\omega}{2E})^2 + \frac{17}{8}\big]. \quad (5)$$

In writing the second term we have used the fact that $E \gg \omega$. This is justified, as for the present problem, partonic jets have very high energy compared to the energy of the plasma constituents which could be $\sim$ 3T. Therefore, this expression and consequently others presented in this paper can be simplified further [17]. Another important difference of Eq.3 and Eq.5 is the appearance of a new term $2E/(2E-\omega)$. This originates from the exchange diagram (u^{-2} term).

The other important reaction is $q\bar{q} \to q\bar{q}$, which also has t^{-2} divergence [19] and therefore, found to contribute significantly to the total energy loss. It should be noted that there is no u^{-2} divergence involved in this process hence the collision is dominated by soft scattering and result do not differ much if the relevant s channel diagram is excluded. We, nevertheless, retain all the diagrams. The differential probability for 'bhabha' like scattering, therefore, takes the form :

$$\Theta_{q\bar{q}\to q\bar{q}}(E,E')dE' = \frac{4}{9}\frac{\pi\alpha_s^2}{\omega E'^2}\rho_q\big[1 - \Delta'\frac{E'}{E} + (2\Delta'-1)\frac{E'^2}{E^2} - \Delta'\frac{E'^3}{E^3} + \frac{E'^4}{E^4}\big]dE' \quad (6)$$

Corresponding energy loss turns out to be :

$$(\frac{dE}{dx})_{q\bar{q}\to q\bar{q}} = \frac{4}{9}\frac{\pi\alpha_s^2}{\omega}\rho_q\big[-\frac{1}{4} - \frac{\Delta'}{3} + \ln\frac{2E}{\omega} + \Delta'\frac{\omega}{2E} - \frac{2\Delta'-1}{2}(\frac{\omega}{2E})^2 + \frac{\Delta'}{3}(\frac{\omega}{2E})^3 - \frac{1}{4}(\frac{\omega}{2E})^4\big], \quad (7)$$

where $\Delta' = 2/3$. The QED limit for the last two equations can be taken by replacing $\frac{2}{9}\alpha_s^2$ with α_{em}^2 and $\Delta' = 2$ [2,3].

Finally we present results for the process involving fusion of quark-antiquark pair to produce double gluons in the final state. This again involved identical particles in the final channel for which appropriate limit is taken. This process is also suppressed because of less sensitive infrared divergences.

$$\Theta_{q\bar{q}\to gg}(E,E')dE' = \frac{4}{3}\frac{\pi\alpha_s^2}{\omega E^2}\rho_q\Big[\frac{4}{9}\{\frac{E}{E'} + \frac{E'}{E-E'}\} + 2\frac{E'}{E} - 2\frac{E'^2}{E^2} - \frac{13}{9}\Big]dE' \quad (8)$$

Other important reactions for which we do not present explicit results include $qq' \to qq'$ and $q\bar{q}' \to q\bar{q}'$. They contribute equally to the energy loss (for matter with $\rho_q = \rho_{\bar{q}}$). It should be mentioned that $q\bar{q} \to q'\bar{q}'$ induced energy loss is small because of the absence of infrared enhancement. This again is less divergent (no t^{-2} or u^{-2}), and, therefore, found to be less effective means of energy dissipation. This also shows up in the expression for dE/dx which does not have any logarithmic energy dependence as evident from :

$$(\frac{dE}{dx})_{q\bar{q}\to gg} = \frac{4}{3}\frac{\pi\alpha_s^2}{\omega}\rho_q\Big[-\frac{53}{288} + \frac{17}{18}(\frac{\omega}{2E})^2 - \frac{2}{3}(\frac{\omega}{2E})^3 + \frac{1}{2}(\frac{\omega}{2E})^4 + \frac{4}{9}\ln 2\Big] \quad (9)$$

Similar to quarks, hard gluons can also dissipate energy while colliding with the plasma constituents. Most important process by which gluons can transfer energy to the plasma is the $gg \to gg$.

$$\Theta_{gg\to gg}(E,E')dE' = \frac{9}{4}\frac{\pi\alpha_s^2}{\omega E^2}\rho_g\Big[3 - \frac{E'(E-E')}{E^2} + \frac{E^2}{E'^2} - \frac{E}{E'} + \frac{EE'}{(E-E')^2}\Big]dE' \quad (10)$$

Corresponding expression for the energy loss can be written as

$$(\frac{dE}{dx})_{gg\to gg} = \frac{9}{4}\frac{\pi\alpha_s^2}{\omega}\rho_g\Big[\frac{451}{192} - 3\ln 2 + \ln(\frac{2E}{\omega}) - \frac{3}{2}(\frac{\omega}{2E})^2 + \frac{1}{3}(\frac{\omega}{2E})^3 - \frac{1}{4}(\frac{\omega}{2E})^4 - \frac{2E}{2E-\omega}\Big] \quad (11)$$

Like quark, the QCD compton scattering also proves to be quite efficient in transferring gluon energy into the plasma. Relevant expressions for the differential collisional probability $\Theta(E,E')dE'$ and dE/dx induced by $gq \to gq$ scattering can be obtained from Eqs. 2 and 3 respectively with ρ_g replaced by ρ_q. It should be mentioned that $gg \to q\bar{q}$ is also suppressed as there is no t^{-2} or u^{-2} singularity involved in this process.

In Fig. 1 we present stopping power as function of energy of the incoming parton at a temperature $T = 250$ MeV. The result is to be compared with previous estimates [1,14]. Evidently, bulk contribution to the total collisional energy loss of quark comes from the $qg \to qg$ channel. Net energy loss of a light quark is given by the sum of all these diagrams including scattering and annihilation processes. Contribution of inelastic channels are found to be small and, therefore, have not been shown explicitly. However, the total loss, as demonstrated in Fig. 1. include effect of all the channels. It should be mentioned that present treatment can be extended for heavy quarks for which collision probabilities will be modified [17]. Quantitatively, we find $dE/dx \sim 0.8$ GeV/fm for a 20 GeV parton, *vis-a-vis* 0.2 GeV/fm of Refs. [1,14,16]. This can be attributed to the diagrams other than t channel.

The results for gluon energy loss is presented in Fig.2 below. Evidently gluon energy loss is mostly driven by $gg \to gg$ scattering. Also comparable is the contribution of $qg \to qg$ channel.

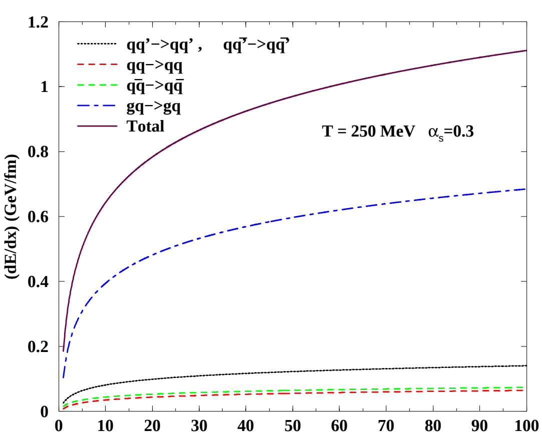

FIG. 1. Individual contributions of various processes responsible for quark energy loss are shown. The aggregated collisional loss is also presented

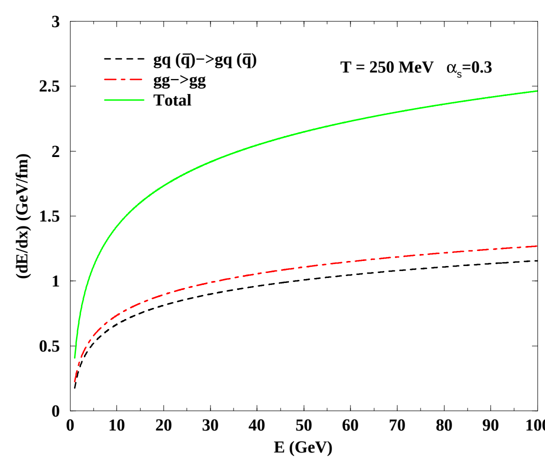

FIG. 2. Individual contributions of various processes responsible for gluon energy loss are shown. Dashed, dot-dashed and solid line represent $gq(\bar{q}) \to gq(\bar{q})$, $gg \to gg$ and total respectively.

To bring the importance of collisional loss into bold relief, we estimate the possible parton density relevant for the RHIC energies. The gluon rapidity density in this case can be taken as $dN_g/dy \sim 1000$, which, when plugged into the Bjorken formula [20] $\rho_g = \frac{dN_g}{dy}/\tau_0 \pi R_{Au}^2$ with formation time $\tau_0 = 0.5$ fm/c, we get a value of $T \sim 400$ MeV. Corresponding values of the energy loss for quark and gluon is significantly large as depicted in Fig 3. It might be mentioned that this density is consistent with the one used in Ref. [6].

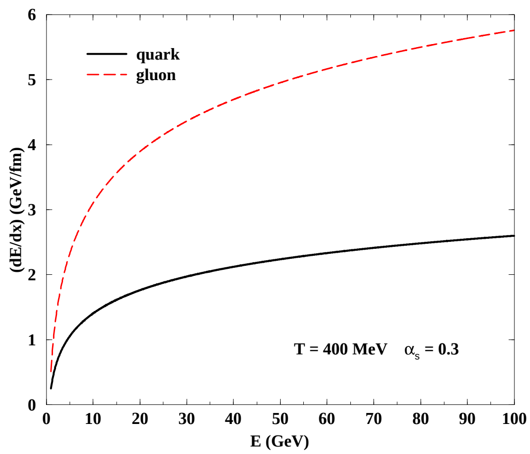

FIG. 3. Individual contributions for quark (solid line) and gluon (dashed line) energy loss.

To summarize, in the present work, we have studied collisional loss of quarks and gluons in a hot partonic medium. Contrary to the previous estimates of refs. [1,14], even at moderate temperature $T \sim 250$ MeV one can achieve considerable energy loss in $2 \to 2$ processes. It is found that $qg \to qg$ is the most dominant mechanism of quark energy dissipation. Even for gluons, this proves to be almost equally effective as gg scattering. We also have presented explicit form of dE/dx for both quarks and gluons. Significant contribution is coming both from u channel and interference terms. Therefore, we conclude that collisional loss is not insignificantly small, hence should be compared to the radiative loss. Furthermore, RHIC data suggests only a tiny amount of 'jet quenching' which, we believe, can be accommodated with the collisional loss if all these channels are taken into account. Such investigations are currently in progress and shall be reported in a subsequent publication [17].

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